

②

$$a_{n+1} = \frac{a_n}{4a_n + 3}$$

$$a_1 = -1$$

$$b_n = \frac{1}{a_n} + 2$$

$$\begin{aligned} \text{b) } \frac{b_{n+1}}{b_n} &= \frac{\frac{1}{a_{n+1}} + 2}{\frac{1}{a_n} + 2} = \frac{\frac{1}{\frac{a_n}{4a_n+3}} + 2}{\frac{1}{a_n} + 2} = \\ &= \frac{\frac{4a_n+3}{a_n} + 2}{\frac{1}{a_n} + 2} = \frac{\frac{6a_n+3}{a_n}}{\frac{1+a_n}{a_n}} = \frac{3(2a_n+1)}{1+a_n} \\ \Rightarrow \boxed{\frac{b_{n+1}}{b_n} = 3} \quad \underline{\text{R.N.}} \quad \Rightarrow \boxed{b_1 = \frac{1}{a_1} + 2 = 1} \end{aligned}$$

$$\text{2) } b_n = \frac{1}{a_n} + 2 \Rightarrow \frac{1}{a_n} = b_n - 2$$

$$+ \frac{1}{a_1} = b_1 - 2$$

$$+ \frac{1}{a_2} = b_2 - 2$$

$$+ \vdots$$

$$+ \frac{1}{a_n} = b_n - 2$$

$$\frac{1}{a_1} + \frac{1}{a_2} + \dots + \frac{1}{a_n} = \underbrace{b_1 + b_2 + \dots + b_n}_{\text{סכום הנקודות}} - 2n = \frac{b_1(2^n - 1)}{2 - 1} - 2n$$

$$= \frac{1 \cdot (3^n - 1)}{3 - 1} - 2n = \frac{3^n - 1}{2} - 2n = \underline{\underline{\frac{3^n - 4n - 1}{2}}}$$

R.N.

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ON - 11

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$$+ \frac{1}{a_3} = b_3 - 2$$

$$\frac{1}{a_n} = b_n - 2$$

$$= \frac{1((-3)^n - 1)}{-3 - 1} = \frac{3^n - 1}{-4} \quad (715 - n)$$

Re. n

$$(3) \quad P(\text{הצלחה}) = p$$

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$$1) \quad \binom{5}{4} \cdot p^4 \cdot (1-p) = 3 \cdot p^5$$

$$5 p^4 (1-p) = 3 p^5 \quad / : p^4$$

$$5(1-p) = 3p$$

$$8p = 5 \Rightarrow \boxed{p = \frac{5}{8}}$$

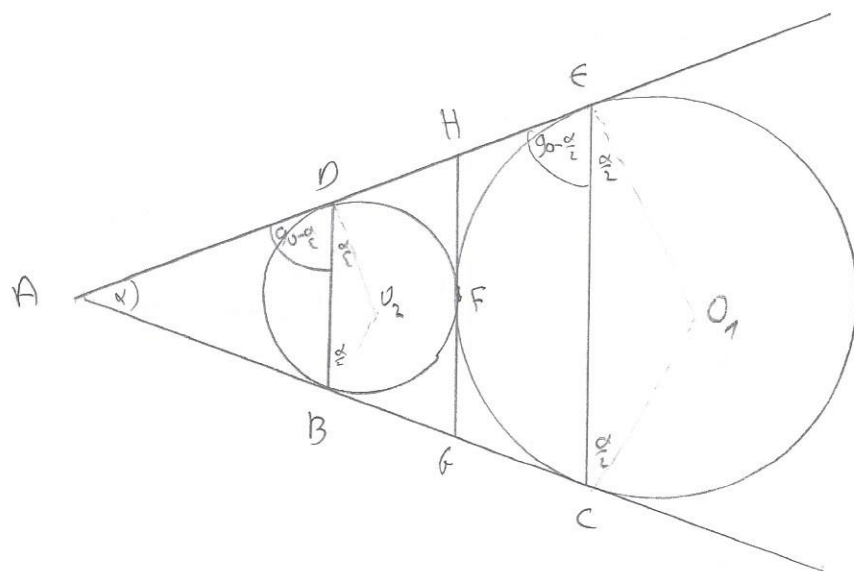
$$2) \quad P(\text{אבדן מלך מלכה}) = \underbrace{\binom{5}{3} \cdot \left(\frac{5}{8}\right)^3 \cdot \left(\frac{3}{8}\right)^2}_{\text{הצלחה בדיון 3-2 נכשלת}} + \underbrace{\binom{5}{4} \cdot \left(\frac{5}{8}\right)^4 \cdot \frac{3}{8}}_{\text{הצלחה בדיון 4 נכשלת}} + \underbrace{\left(\frac{5}{8}\right)^5}_{\text{הצלחה בדיון 5}} = \underline{\underline{0.724}}$$

$$3) \quad P(\text{אבדן מלך מלכה}) = \underbrace{\binom{5}{3} \cdot \left(\frac{5}{8}\right)^3 \cdot \frac{3}{8}}_{\text{הצלחה בדיון 3-2 נכשלת}} + \underbrace{\left(\frac{5}{8}\right)^4}_{\text{הצלחה בדיון 4 נכשלת}} = \underline{\underline{0.518}}$$

$$4) \quad P(\text{הצלחה}) = \frac{5}{8}$$

$$P(\text{הצלחה}) = \underline{\underline{0.518}} \leftarrow$$

אין שאלה ראשונה
הצלחה בדיון 5



- 1) AC מלבן $ABCD$ (נתון)
- 2) AE מלבן $ABCD$ (נתון)
- 3) $\angle A = 90^\circ$ (היכוד)
- 4) $AE = AC$ (נתון: $AE = AC$ ו- $AC = 2$, $AE = 2$)
- 5) $\angle AEC = \angle ACE = 90^\circ - \frac{\alpha}{2}$ (נתון: $\angle AEC = \angle ACE$ ו- $\angle AEC = 90^\circ - \frac{\alpha}{2}$)
- 6) $AD = AB$ (נתון: $AD = AB$ ו- $AD = 2$, $AB = 2$)
- 7) $\angle ADB = \angle ABD = 90^\circ - \frac{\alpha}{2}$ (נתון: $\angle ADB = \angle ABD$ ו- $\angle ADB = 90^\circ - \frac{\alpha}{2}$)
- 8) $BC \parallel DE$ (נתון: $BC \parallel DE$ ו- $BC = 5$, $DE = 5$)
- 9) $DE \perp BC$ (נתון: $DE \perp BC$ ו- $DE = 5$, $BC = 5$)
- 10) $DECB$ (נתון: $DECB$ ו- $DECB$ ו- $DECB$ ו- $DECB$)
- 11) $DECB$ (נתון: $DECB$ ו- $DECB$ ו- $DECB$ ו- $DECB$)

12) $\{H, D, E, F, G, H\}$

117)

13) $HD = HF$ ($\hookrightarrow$ 100 שאלות + 12, 1 נקודות)

14) $HE = HF$ ($\hookrightarrow$ 100 שאלות + 12, 1 נקודות)

15) $\boxed{HD = HE}$ (נקודות 100 + 14, 13 נקודות)

16) $GB = GF$ ($\hookrightarrow$ 100 שאלות + 12, 2 נקודות)

17) $GC = GF$ ($\downarrow$
- " -)

18) $\boxed{GB = GC}$ (נקודות 100 + 17, 16 נקודות)

19) $\boxed{\begin{matrix} \text{שאלות 100} \\ \text{נקודות 18} \end{matrix} \quad GH}$ (18, 15 נקודות)
R.N

20) $\angle O_1EA = \angle O_2DB = 90^\circ$ (2 נקודות + 2 נקודות נוספות)

21) $\angle O_1EC = \frac{\alpha}{2}$ (5 נקודות + 20, 5 נקודות)

22) $\angle O_1EC = \angle O_1CE = \frac{\alpha}{2}$ (21 נקודות + 20, 5 נקודות)

23) $\angle O_2DB = \frac{\alpha}{2}$ (7 נקודות + 20, 7 נקודות)

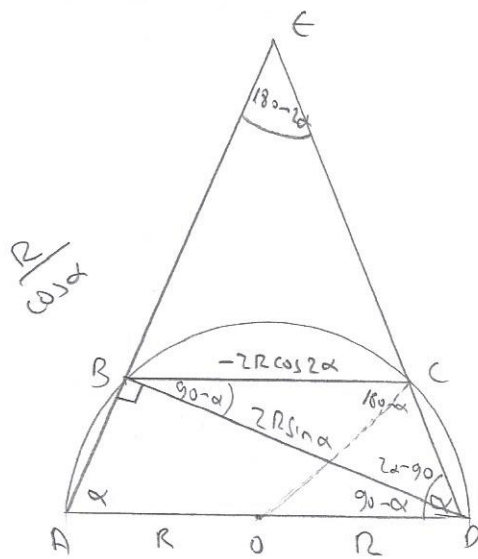
24) $\angle O_2DB = \angle O_2BD = \frac{\alpha}{2}$ (22 שאלות + 23 נקודות)

25) $\boxed{\Delta O_1EC \sim \Delta O_2DB}$ (22 שאלות + 24, 22 נקודות)

26) $\frac{O_1E}{EC} = \frac{O_2D}{BD}$ ($\hookrightarrow$ נקודות 100)

$\Rightarrow O_1E \cdot BD = O_2D \cdot EC$ $\begin{cases} O_1E = R \\ O_2D = r \end{cases}$

$\Rightarrow \boxed{R \cdot BD = r \cdot EC}$ R.N



- 1) מרכז ABC (O) (BC || AD) (נתון)
מרכז הטרפז
- 2) BD בנ"ח
- 3) $\angle CBD = \angle BDA$ (זווית 1 + 5, להחליף בן ישרים מקבילים)
- 4) $AB = CD$ (זווית 3 + 5, הקדמנו שווה, משולשים ישרים)
- 5) $\angle EAD = \angle EDA = \alpha$ (זווית 4, 5, בדקנו שווה שווה)
(זווית 1 + 5, 9)
- 6) $AO = OD = R$ (נתון)
- 7) $\angle ABD = 90^\circ$ (זווית 6 + 5, זווית הקדמון ישרה)
- 8) $\angle BDA = 90 - \alpha$ (זווית 5, 7, סכום 5 ב $\triangle ABD$ 180)
- 9) $\angle CBD = \angle BDA = 90 - \alpha$ (זווית 1, 8, להחליף בן ישרים מקבילים)
- 10) $\angle CDB = 2\alpha - 90$ (זווית 5, 9, ישר זווית)
- 11) $\angle BCD = 180 - \alpha$ (זווית 9, 10, סכום 5 ב $\triangle BCD$ 180)
- 12) $\triangle ABD$

$$\frac{BD}{2R} = \sin \alpha \Rightarrow \boxed{BD = 2R \sin \alpha}$$



13) ΔBCD

$$\frac{BC}{\sin(2\alpha - 90)} = \frac{2R \sin \alpha}{\sin(180 - \alpha)}$$

$-\cos 2\alpha$ $\sin \alpha$

$$\Rightarrow BC = -\frac{2R \sin \alpha \cdot \cos 2\alpha}{\sin \alpha}$$

$$\boxed{BC = -2R \cos 2\alpha}$$

1/2 Rnd

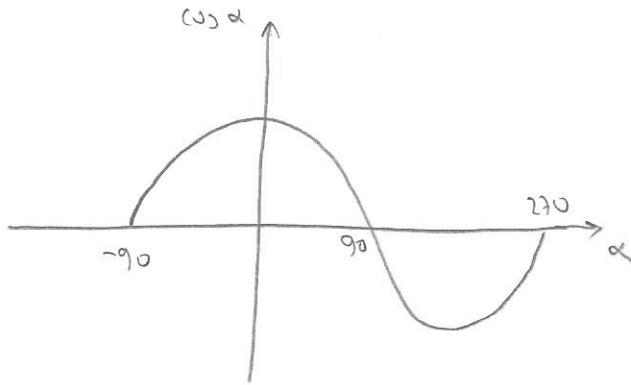
2) $BC > 0$

2nd

$0 < \alpha < 90$ (ΔABC 1st)

$$-2R \cos 2\alpha > 0$$

$$\Rightarrow \cos 2\alpha < 0$$



$\Downarrow$

$$90 < 2\alpha < 270$$

$$\boxed{45 < \alpha < 135}$$

2nd

$$\boxed{0 < \alpha < 90}$$

$\Downarrow$

$$\boxed{45 < \alpha < 90}$$

1/2 Rnd

$\leftarrow$

$$14) OC = OD = R \quad (11n) + 6 \text{ 'dp'}$$

$$15) \angle COD = \angle ODC = \alpha \quad (\Delta ODC \sim \Delta OAC \text{ PN + 14, 5 'dp'})$$

11n 5 11/11

$$16) \angle COD = 180 - 2\alpha \quad (180^\circ \Delta COD \sim \Delta OAC \text{ PN + 15 'dp'})$$

$$17) S_{\Delta COD} = \frac{R^2 \cdot \sin(180 - 2\alpha)}{2} = \frac{R^2 \cdot \sin 2\alpha}{2}$$

$$18) \angle E = 180 - 2\alpha \quad (180^\circ \Delta AED \sim \Delta OAC \text{ PN + 5 'dp'})$$

$$19) \Delta AED$$

$$\frac{2R}{\sin(180 - 2\alpha)} = \frac{AE}{\sin \alpha} \Rightarrow AE = \frac{2R \cdot \sin \alpha}{\sin 2\alpha} = \frac{2R \cdot \sin \alpha}{2 \sin \alpha \cdot \cos \alpha}$$

$$\boxed{AE = \frac{R}{\cos \alpha}}$$

$$20) S_{\Delta AED} = \frac{\frac{R}{\cos \alpha} \cdot 2R \cdot \sin \alpha}{2} \Rightarrow \boxed{S_{\Delta AED} = \frac{R^2 \cdot \sin \alpha}{\cos \alpha}}$$

$$21) \frac{S_{\Delta AED}}{S_{\Delta COD}} = 9 \quad (11n)$$

$$\Rightarrow \frac{\frac{R^2 \sin \alpha}{\cos \alpha}}{\frac{R^2 \sin 2\alpha}{2}} = 9 \Rightarrow \frac{2 \sin \alpha}{2 \sin \alpha \cdot \cos^2 \alpha} = 9$$

$$\Rightarrow \cos^2 \alpha = \frac{1}{9} \Rightarrow \cos \alpha = \pm \frac{1}{3}$$

$$\cos \alpha = \frac{1}{3}$$

$$\alpha = \pm 70.52^\circ + 360^\circ k$$

$$\cos \alpha = -\frac{1}{3}$$

$$\alpha = \pm 109.47^\circ + 360^\circ k$$

$$45 < \alpha < 90 \quad (\text{'s for 'dp'})$$

$$\Rightarrow \boxed{\alpha = 70.52^\circ}$$

←

ΔAED

$$\frac{\cancel{r}R}{\sin(180-2\alpha)} = \cancel{r}r$$

(point from origin - r)
 ΔAED is

$$\Rightarrow \boxed{r = \frac{R}{\sin 2\alpha}}$$

$$\frac{r}{R} = \frac{\cancel{R}}{\sin 2\alpha} = \frac{1}{\sin 2\alpha} = \frac{1}{\sin 41.05}$$

$$\Rightarrow \boxed{\frac{r}{R} = 1.59}$$

$$(6) \quad f(x) = \frac{ax^2 + 4x}{x^2 + 3x + b}$$

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נבדוק $x=1$ נכון

$$\Rightarrow 1^2 + 3 \cdot 1 + b = 0 \Rightarrow \boxed{b = -4}$$

$$f(x) = \frac{ax^2 + 4x}{x^2 + 3x - 4}$$

נבדוק $y=1$ נכון

$$\begin{aligned} \lim_{x \rightarrow \pm\infty} \frac{ax^2 + 4x}{x^2 + 3x - 4} &= \lim_{x \rightarrow \pm\infty} \frac{a + \frac{4}{x}}{1 + \frac{3}{x} - \frac{4}{x^2}} = \frac{a + \frac{4}{\infty}}{1 + \frac{3}{\infty} - \frac{4}{\infty^2}} = \\ &= a \Rightarrow \boxed{a = 1} \Rightarrow \boxed{\lim_{x \rightarrow \pm\infty} y = 1} \end{aligned}$$

$$f(x) = \frac{x^2 + 4x}{x^2 + 3x - 4}$$

2) 1) : נדרש

$$x^2 + 3x - 4 \neq 0$$

$$\boxed{\begin{matrix} x \neq -4 \\ x \neq 1 \end{matrix}} \quad \underline{\text{ק"נ}}$$

2) אילו נקודות

$$y=0 \Rightarrow x^2 + 4x = 0 \Rightarrow x(x+4) = 0 \Rightarrow \boxed{x=0}$$

$$\boxed{(0, 0)}$$

$x=-4$ ϕ (נכנס)
נדרש

אילו נקודות

$$x=0 \Rightarrow f(0) = 0 \Rightarrow \boxed{(0, 0)}$$



3)

מאחר ש $x = -4$ הוא נקודה חלולה

$$\lim_{x \rightarrow -4} \frac{x^2 + 4x}{x^2 + 3x - 4} = \lim_{x \rightarrow -4} \frac{x(x+4)}{(x+4)(x-1)} = \lim_{x \rightarrow -4} \frac{x}{x-1} = \frac{-4}{-5} = \frac{4}{5}$$

נקודה חלולה: $(-4, \frac{4}{5})$

נקודה חלולה: $(-4, \frac{4}{5})$

$$f(x) = \frac{x}{x-1} \quad x \neq 1, -4$$

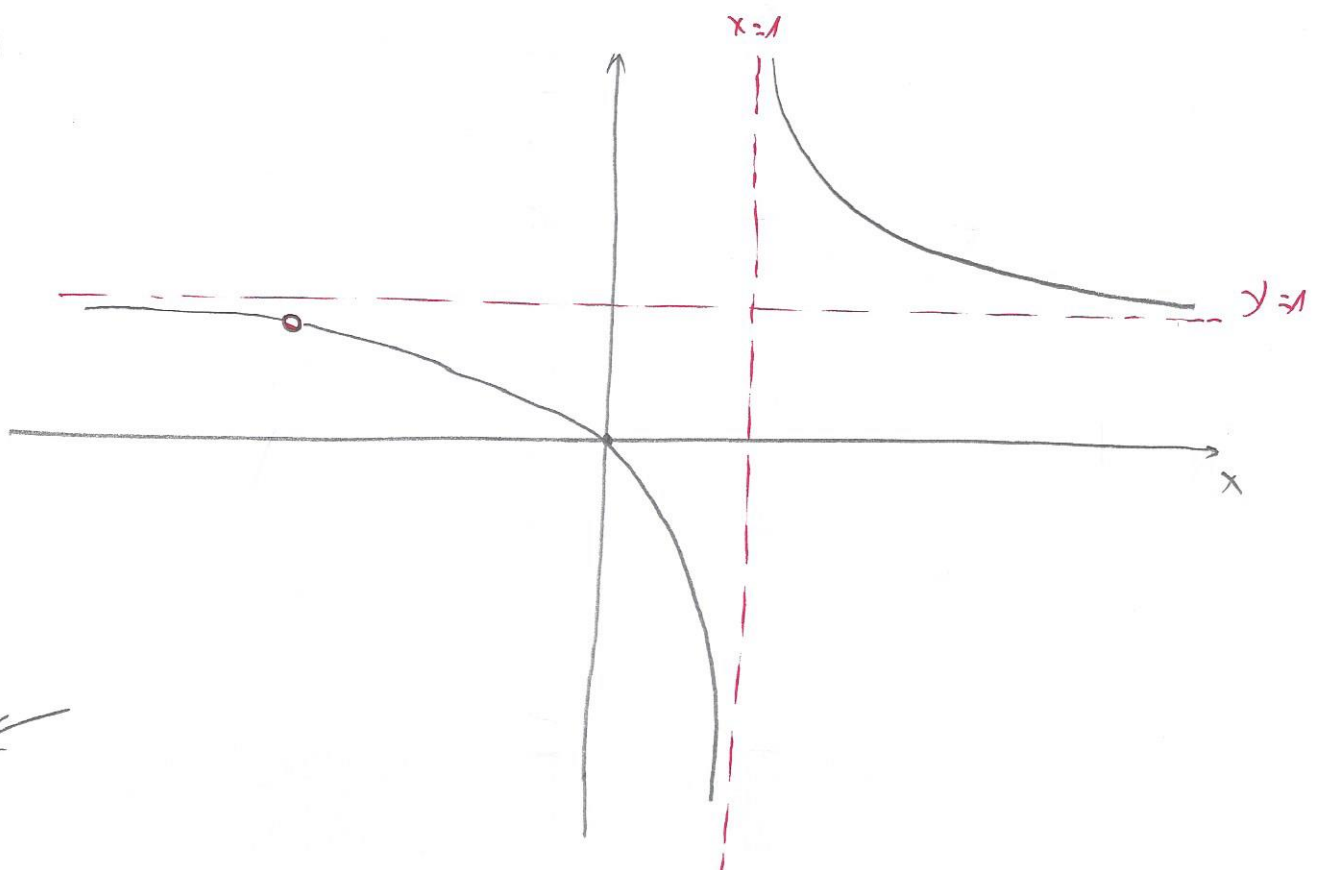
4)

נגזרת

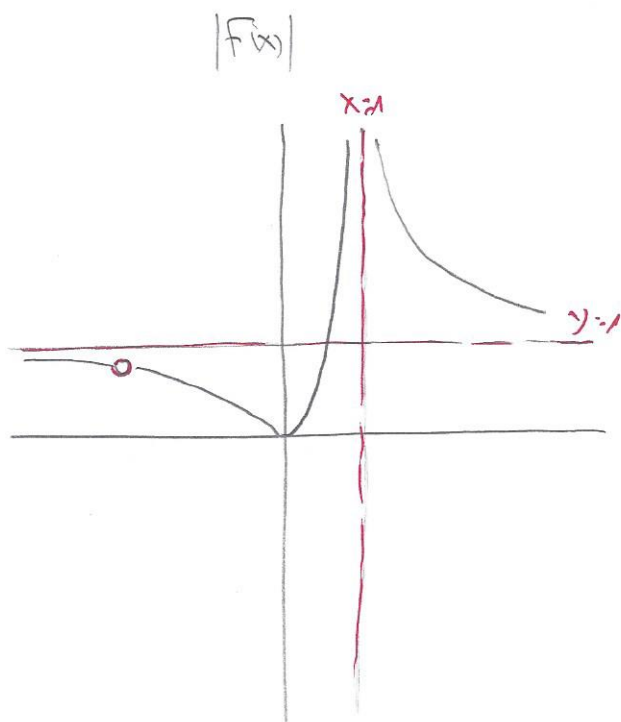
$$f'(x) = \frac{x-1-x}{(x-1)^2} \Rightarrow f'(x) = -\frac{1}{(x-1)^2}$$

הנגזרת (סימן) של $f(x)$ היא $f'(x) = -\frac{1}{(x-1)^2}$.
הנגזרת של $f(x)$ היא $f'(x) = -\frac{1}{(x-1)^2}$.

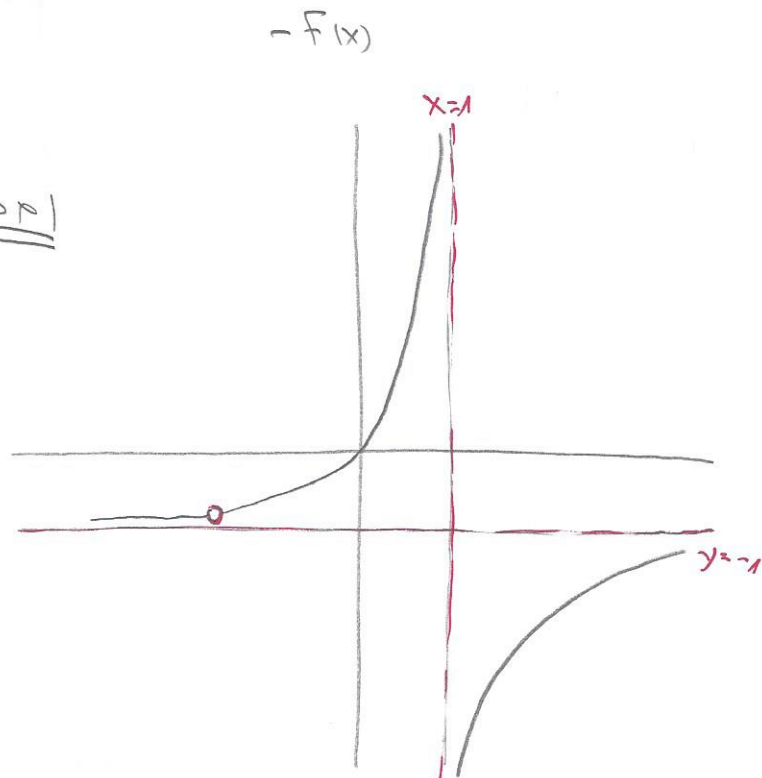
5)



3) $|f(x)| = -f(x)$



||p||



$$0 \leq x < 1$$

$$|f(x)| \geq$$

$$x \neq -1, 1$$

||p||

$$-f(x) \geq 0$$

$$0 \leq x < 1$$

||p||

$$0 \leq x < 1$$

4) $g(x) = f^2(x) \cdot \hat{f}(x)$

$$x \rightarrow 1^- \Rightarrow \lim_{x \rightarrow 1^-} g(x)$$

$$y=0 \Rightarrow f^2(x) \cdot \hat{f}(x) = 0$$

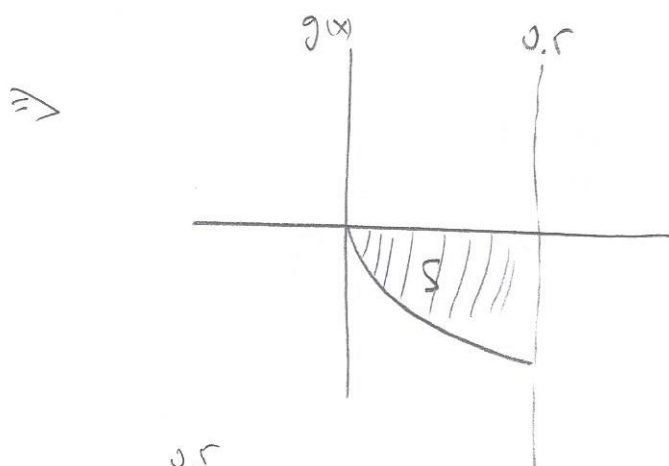
$$f(x)=0 \Rightarrow (0,0)$$

$$\hat{f}(x)=0 \Rightarrow \emptyset$$



$$g(0.5) = \underset{\vee}{F(0.5)} \cdot \underset{\wedge}{F'(0.5)} < 0$$

be. cor. 2.1



$$S = \int_0^{0.5} -F^2(x) \cdot F'(x) dx =$$

$$\left\{ u = F(x) \Rightarrow du = F'(x) dx \right\}$$

$$\int_0^{0.5} -u^2 \cdot du = \left[-\frac{u^3}{3} \right]_0^{0.5} = - \left[\frac{F^3(x)}{3} \right]_0^{0.5}$$

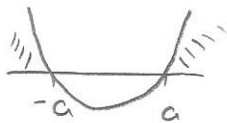
$$= -\frac{F^3(0.5)}{3} - \left[-\frac{F^3(0)}{3} \right] = -\frac{F^3(0.5)}{3} = -\frac{\left(\frac{0.5}{0.5-1}\right)^3}{3} = \underline{\underline{\frac{1}{3}}}$$

Result

7)
$$f(x) = \frac{x}{\sqrt{x^2 - a^2}}$$

$(a > 0)$

6) $x^2 - a^2 > 0$



$$x < -a \quad \text{or} \quad x > a$$

7) נציק נגזרת

$$x^2 - a^2 = 0 \Rightarrow x^2 = a^2 \Rightarrow x = \pm a$$

נציק נגזרת

$$\lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2 - a^2}} = \frac{\infty}{\sqrt{\infty^2 - a^2}} = \frac{\infty}{\infty} = 1$$

$$x \rightarrow \infty$$

$$y = 1$$

$$\lim_{x \rightarrow -\infty} \frac{x}{\sqrt{x^2 - a^2}} = \frac{-\infty}{\sqrt{\infty^2 - a^2}} = \frac{-\infty}{\infty} = -1$$

$$x \rightarrow -\infty$$

$$y = -1$$

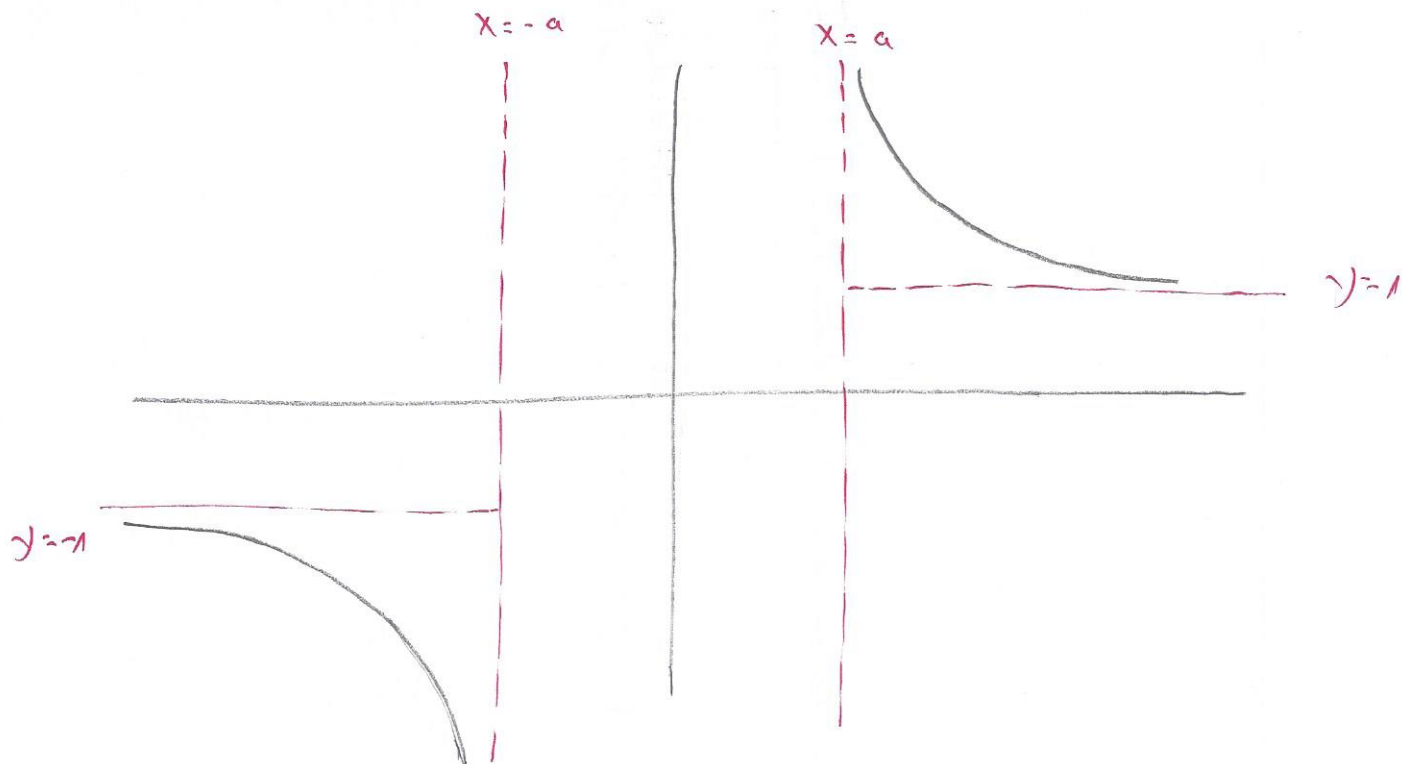
8)
$$f'(x) = \frac{\sqrt{x^2 - a^2} - x \cdot \frac{x}{\sqrt{x^2 - a^2}}}{x^2 - a^2} = \frac{x^2 - a^2 - x^2}{(x^2 - a^2) \cdot \sqrt{x^2 - a^2}}$$

$$f'(x) = -\frac{a^2}{(x^2 - a^2) \cdot \sqrt{x^2 - a^2}}$$

$a > 0 \Rightarrow f'(x) < 0$ x גדל
הפונקציה יורדת



$$\text{כל } x \text{ שבה } x > a \text{ או } x < -a$$



7) 1)

Nullstellen

$$x^2 - a^2 = 0 \Rightarrow \boxed{x = \pm a}$$

Asymptoten

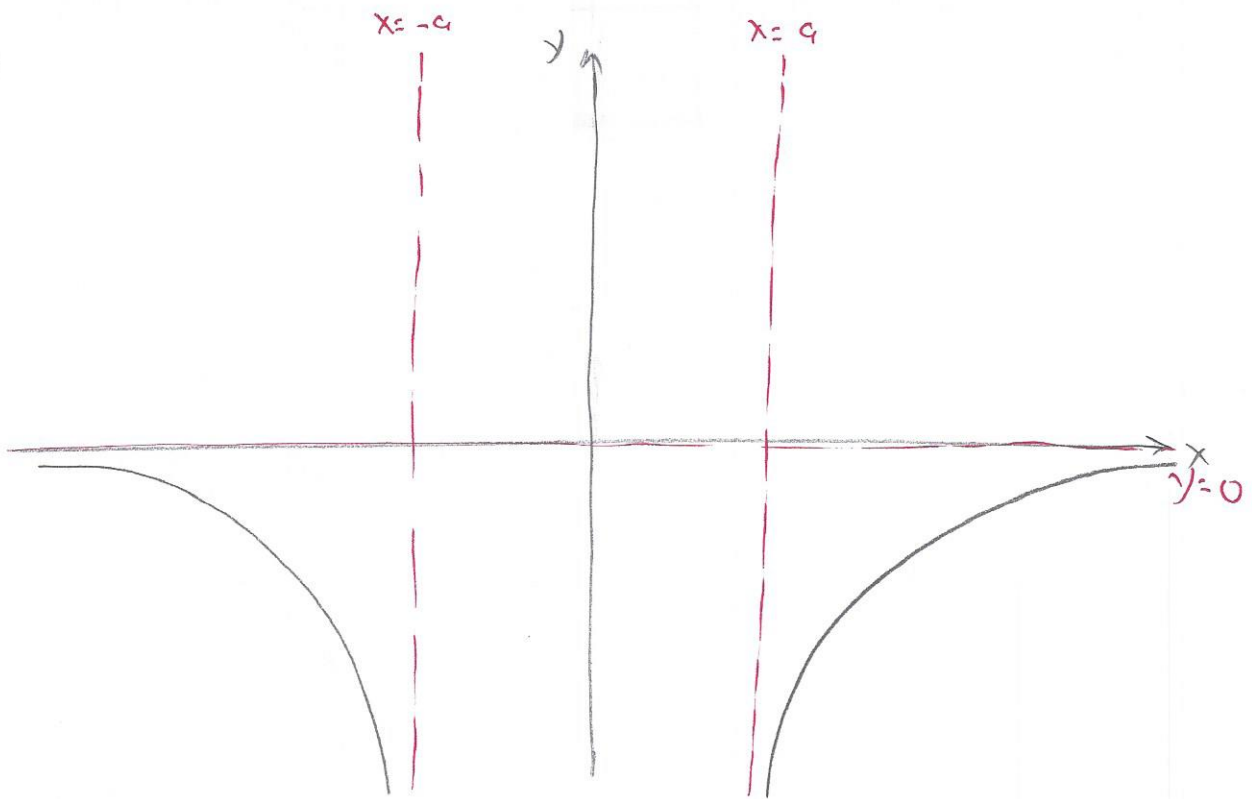
$$f(x) = -\frac{a^2}{(x^2 - a^2)^{\frac{3}{2}}}$$

$$\lim_{x \rightarrow \pm \infty} -\frac{a^2}{(x^2 - a^2)^{\frac{3}{2}}} \cdot \frac{x^3}{x^3} = \lim_{x \rightarrow \pm \infty} -\frac{\frac{a^2}{x^3}}{\left(1 - \frac{a^2}{x^2}\right)^{\frac{3}{2}}} = -\frac{\frac{a^2}{\infty^3}}{\left(1 - \frac{0^2}{\infty^2}\right)^{\frac{3}{2}}} = 0$$

$$= \frac{0}{1} = 0 \Rightarrow \boxed{\lim_{x \rightarrow \pm \infty} y = 0}$$



2)



$$1) \int_{2a}^{3a} f(x) dx + \int_{-3a}^{-2a} f(x) dx$$

$$f(-x) = \frac{-x}{\sqrt{x^2 - a^2}} \Rightarrow -f(-x) = \frac{x}{\sqrt{x^2 - a^2}} \quad \text{"/u 17}$$

$$\Rightarrow \boxed{f(x) = -f(-x)} \rightarrow \text{"/u 17}$$

$$\int_{2a}^{3a} f(x) dx = - \int_{-3a}^{-2a} f(x) dx$$

$$\Rightarrow \boxed{\int_{2a}^{3a} f(x) dx + \int_{-3a}^{-2a} f(x) dx = 0}$$

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$$\int_{2a}^{3a} \frac{x}{\sqrt{x^2 - a^2}} dx = \left\{ \begin{array}{l} u = x^2 - a^2 \Rightarrow du = 2x dx \\ \Rightarrow dx = \frac{du}{2x} \end{array} \right.$$

$$\int_{2a}^{3a} \frac{x}{\sqrt{u}} \cdot \frac{du}{2x} = \int_{2a}^{3a} \frac{u^{-\frac{1}{2}}}{2} du = \left[\frac{u^{\frac{1}{2}}}{\frac{1}{2} \cdot 2} \right]_{2a}^{3a}$$

$$= \left[\sqrt{u} \right]_{2a}^{3a} = \left[\sqrt{x^2 - a^2} \right]_{2a}^{3a} = \underline{\underline{\sqrt{8}a - \sqrt{3}a}}$$

$$\int_{-3a}^{-2a} \frac{x}{\sqrt{x^2 - a^2}} dx = \left[\sqrt{x^2 - a^2} \right]_{-3a}^{-2a} = \underline{\underline{\sqrt{3}a - \sqrt{8}a}}$$

$$\Rightarrow \int_{2a}^{3a} f(x) dx + \int_{-3a}^{-2a} f(x) dx = 0$$

2) a = 0

$$f(x) = \frac{x}{\sqrt{x^2}} = \frac{x}{|x|}$$

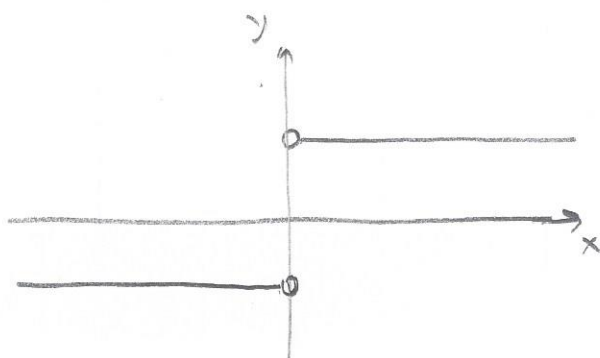
1) : 2 113 11

x ≠ 0

2) $x > 0 \Rightarrow f(x) = 1$

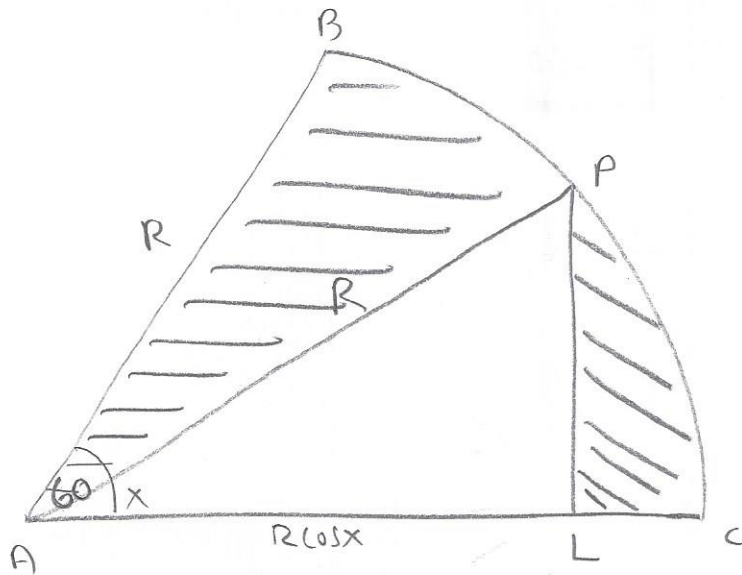
$x < 0 \Rightarrow f(x) = -1$

$\Rightarrow$



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* $A = 60^\circ$

($\frac{360}{6} = 60^\circ$ (for $\frac{1}{6}$ of circle))

* $\angle PAL = x$

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DPAL

$$\frac{AL}{R} = \cos x \Rightarrow AL = R \cdot \cos x$$

$$S_{DAFL} = \frac{R \cdot R \cos x \cdot \sin x}{2} = \frac{R^2 \sin 2x}{4}$$

$$F(x) = \frac{1}{6} \pi R^2 - \frac{R^2 \sin 2x}{4}$$

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$$\hat{F}(x) = -\frac{R^2}{2} \cos 2x$$

$$\hat{F}(x) = 0 \Rightarrow \cos 2x = 0 \Rightarrow 2x = \frac{\pi}{2} + \pi k \quad / : 2$$

$$x = \frac{\pi}{4} + \frac{\pi}{2} k$$

$k=0$ for $\Delta = 15^\circ - x$

$$x = \frac{\pi}{4}$$



$$F(x) = R^2 \sin 2x$$

$$F'(x) = R^2 \cdot \sin 90 > 0 \Rightarrow \boxed{\min x = \frac{\pi}{4}} \quad \text{r.k.}$$

$$2) F\left(\frac{\pi}{4}\right) = 24\pi - 36 \quad | \text{in}$$

$$\Rightarrow \frac{1}{6} \pi R^2 - \frac{R^2 \cdot \sin 90}{4} = 24\pi - 36$$

$$\frac{1}{6} \pi R^2 - \frac{R^2}{4} = 24\pi - 36$$

$$R^2 \left(\frac{\pi}{6} - \frac{1}{4} \right) = 24\pi - 36 \Rightarrow R^2 \left(\frac{2\pi - 3}{12} \right) = 24\pi - 36$$

$$R^2 = \frac{12(24\pi - 36)}{2\pi - 3} = \frac{144(2\pi - 3)}{2\pi - 3} = 144$$

$$R > 0 \Rightarrow \boxed{R = 12} \quad \text{r.k.}$$

$$\Rightarrow \text{הטח המקסימלית הליניארית} \quad \text{כבר}$$

$$x = \frac{\pi}{4}$$

$$\Downarrow$$

הטח המקסימלית הליניארית

$$x = \frac{\pi}{4}$$

$$\boxed{S_{\text{APL}} = \frac{R^2 \cdot \sin 90}{4} = \frac{144}{4} = 36}$$

r.k.